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NON-COMMUTATIVE CLARKSON INEQUALITIES FOR SYMMETRIC SPACE

Lemma 1. *Let x be a positive τ-measurable operator.*

(i) If f is a convex function on $[0,\infty)$, then

$$f(\langle x\xi,\xi\rangle) \leqslant \langle f(x)\xi,\xi\rangle$$

for every unit vector ξ in $D(x)$.

(ii) If g is a concave function on $[0,\infty)$, then

$$\langle g(x)\xi,\xi\rangle \leqslant g(\langle x\xi,\xi\rangle)$$

for every unit vector ξ in $D(x)$.

Lemma 2. *Let x and y be positive τ-measurable operators and let E be an exact interpolation space for the couple $(L_1(0,\infty), L_\infty(0,\infty))$.*

(i) If f is a non-negative operator convex function on $[0,\infty)$ with $f(0)=0$, then

$$\|f(x)+f(y)\|_{E(\mathcal{M})} \leqslant 2\|f(x+y)\|_{E(\mathcal{M})}.$$

(ii) If g is a non-negative increasing continuous concave function on $[0,\infty)$, then

$$\|g(x+y)\|_{E(\mathcal{M})} \leqslant 4\|g(x)+g(y)\|_{E(\mathcal{M})}.$$

Theorem 1. *Let x and y be τ-measurable operators and let f be an increasing continuous function on $[0,\infty)$ such that $f(0)=0$ and $g(t)=f(\sqrt{t})$ is operator convex. Then*

$$\begin{aligned}\|f(|x|)+f(|y|)\|_{E(\mathcal{M})} &\leqslant \|f(|x+y|)+f(|x-y|)\|_{E(\mathcal{M})}\\ &\leqslant \|f(2|x|)+f(2|y|)\|_{E(\mathcal{M})}.\end{aligned}$$

Theorem 2. *Let x and y be τ-measurable operators and let f be a nonnegative increasing continuous function on $[0,\infty)$ such that $h(t)=f(\sqrt{t})$ is concave. Then*

$$\begin{aligned}\tfrac{1}{8}\|f(2|x|)+f(2|y|)\|_{E(\mathcal{M})} &\leqslant \|f(|x+y|)+f(|x-y|)\|_{E(\mathcal{M})}\\ &\leqslant 8\|f(|x|)+f(|y|)\|_{E(\mathcal{M})}.\end{aligned}$$

Corollary 1. *Let x and y be τ measurable operators. Then*

$$\|\,|x|^p+|y|^p\|_{E(\mathcal{M})}\leqslant\|\,|x+y|^p+|x-y|^p\|_{E(\mathcal{M})}\leqslant 2^p\|\,|x|^p+|y|^p\|_{E(\mathcal{M})}.$$

for $2\leqslant p\leqslant 4$, and

$$2^{p-3}\|\,|x|^p+|y|^p\|_{E(\mathcal{M})}\leqslant\|\,|x+y|^p+|x-y|^p\|_{E(\mathcal{M})}\leqslant$$

$$\leqslant 8\|\,|x|^p+|y|^p\|_{E(\mathcal{M})}.$$

for $0\leqslant p\leqslant 2$.

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